MEC-13
Control Charts

<u>Outline</u>

- Basic Terms
- Types of Control Charts with their purpose
- Creating Control Charts

<u>Readings</u>

- PMBoK, page 238.
- Rita, pages 305-308.
- Reid, Ch 6.
- Krajewski. Ch 6.
- Lecture Notes.
- Class Notes.

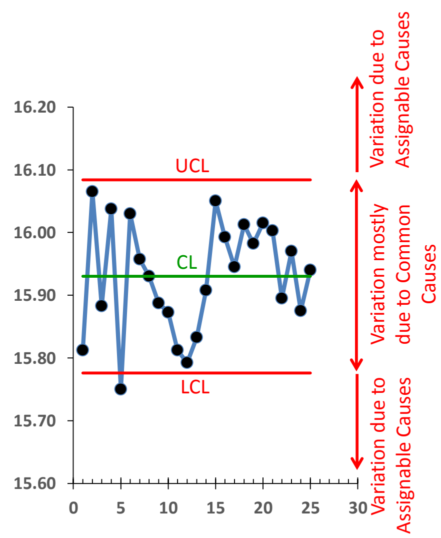
Causes of Variation

Common Causes (aka Random Var):

- Random causes that cannot be identified
- Unavoidable
- Causes like fair wear and tear of machinery, weather changes etc cause slight differences in process variables like diameter, weight, service time, temperature, etc.

Assignable Causes:

- Causes that can be identified and eliminated
- Typical causes are poor employee training, worn tool, machine needing repairs, etc.



Types of Data or Characteristics

Attribute or Discrete Data/Characteristics:

Data or characteristics that has a discrete value, cannot be measured and can only be counted, like good or rotten apples, Yes or No answers, number of people, number of broken cookies in a packet, etc

Variable or Continuous Data:

Data or characteristics that can be measured and has a continuum of values, like height, weight, volume etc

Defects & Defectives

Defect (or Non-Conformity):

- A non-conforming quality characteristic, or a non-conformity, on an item
- Examples: Number of scratches on a tile, number of complaints received, number of bacteria in a petri dish, number of barnacles on the bottom of a boat, number of paint blemishes on auto body observed for an 'x' number of cars, number of imperfections in bond paper – by area inspected and number of imperfections etc
- Item generally Useable
- Charts used: c & u Charts

Defective (or Non-Conforming Unit):

- An item having one or more defects
- Examples: a carton with an x number of broken eggs, a packet with an x number of broken tiles; number of samples out of 20 with x,y,z number of nonconforming cables, number of packets with x,y,z number of nonconforming floppy disks when testing a sample of 200 for 25 trials etc
- Item generally Not Useable
- Charts used: n & np Charts

Control Charts

Control Charts:

- are graphic displays of process data over time and against established control limits, which has a centerline that assists in detecting a trend or run of plotted values toward the either control limit, or change of dispersion either side of the centerline.
- in short, determine whether or not a process is stable or has predictable performance

Control Charts in Project Management

- Although used most frequently to track repetitive activities required for producing manufactured lots, control charts may also be used on projects to monitor various types of output variables, like the cost and schedule variances, volume, and frequency of scope changes, or other management results to help determine if the project management processes are in control
- The project manager and appropriate stakeholders may use the statistically calculated control limits to identify the points at which corrective action will be taken to prevent unnatural performance
- The corrective action typically seeks to maintain the natural stability of a stable and capable process, within the UCL and LCL

Upper & Lower Specification Limits (USL & LSL)

- Maximum and Minimum, allowable or acceptable, performance values agreed between the buyer and the seller, and made part of the agreement
- Also known as the "Voice of the Customer"
- There may be penalties associated with exceeding the ULS or falling below the LSL

Upper & Lower Control Limits (UCL & LCL)

- UCL and LCL are different from the USL and LSL
- Also known as the "Voice of the Process"
- Determined using standard statistical calculations and principles to ultimately establish the natural capability for a stable process
- For repetitive processes, the control limits are generally set at $\pm$ 3 standard deviations around a process mean that has been set at 0 standard deviation
- A process is considered out of control when:
  - a data point exceeds a control limit;
  - seven consecutive plot points are above the mean; or
  - seven consecutive plot points are below the mean.

Basic Types of Variations

Run or Shift:

- Describes the situation when seven or more consecutive points occur on one side of the center line
- Indicates that a special cause has influenced the process
- Points on the center line don't count; they neither break the string, nor add to it.

Trend:

- Describes the situation when 6-7 consecutive points occur in the same direction towards the UCL or LCL
- Indicates that a special cause is acting on the process to cause a trend. Flat line segments don't count, either to break a trend, or to count towards it

Standard Deviation (SD or σ)

- A measure that is used to quantify the amount of Variation or Dispersion of a set of data values

  Low SD $\Rightarrow$ Data points tend to be close to the mean (also called the expected value) of the set

  High SD $\Rightarrow$ Data points spread out over a wider range of values

- Example:

  Data Set: 2,4,4,4,5,5,7,9

  Mean = (2+4+4+4+5+5+7+9)/8 = 40/8 = 5

  Total Variance = $(5-2)^2+(5-4)^2+(5-4)^2+(5-4)^2+(5-5)^2+(5-5)^2+(5-7)^2+(5-9)^2$ = 32

  Mean Variance = 32/8 = 4 = σ^2

  SD = $\sigma = \sqrt{4} = 2$

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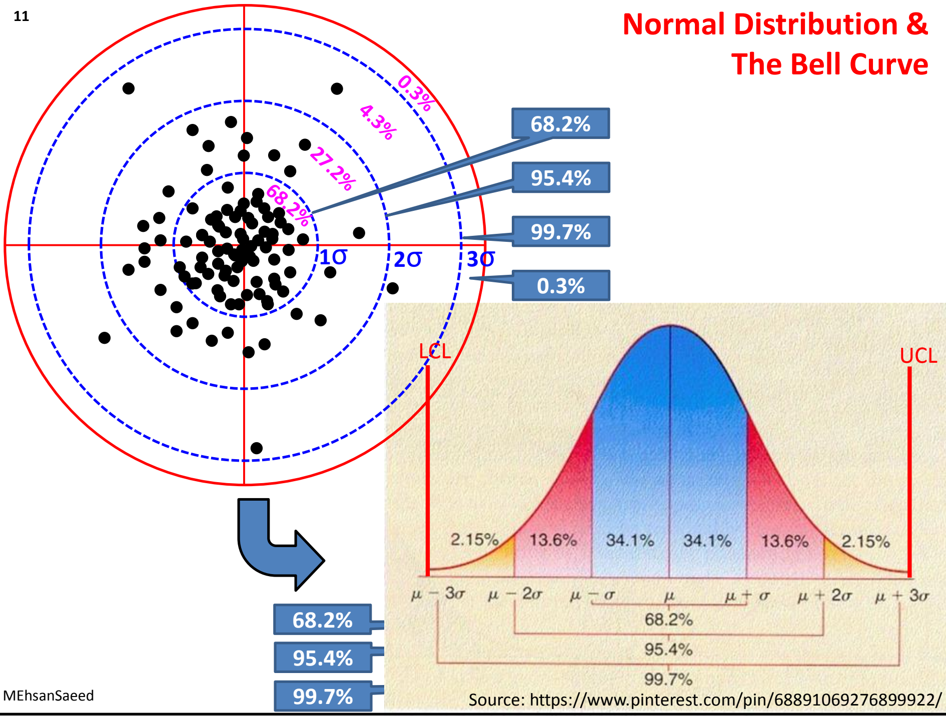
Normal Distribution & The Bell Curve



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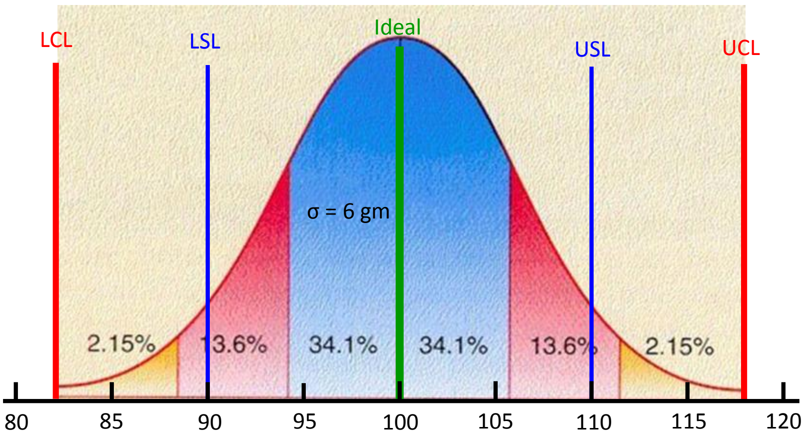
Normal Distribution must lie within Specification Limits – 1/2

Say a deliverable is to manufactured at 100 gm within ± 10 gm ie USL=110 & LSL=90. For the production process to support the Specification Limits (SLs), the entire process must lie within the SLs. Otherwise many deliverables will not meet the specifications and will be rejected by own QC and the customer. In this example, clearly the process, which is producing at σ = 6 gm, is incapable of meeting the SLs, because CLs are outside the SLs



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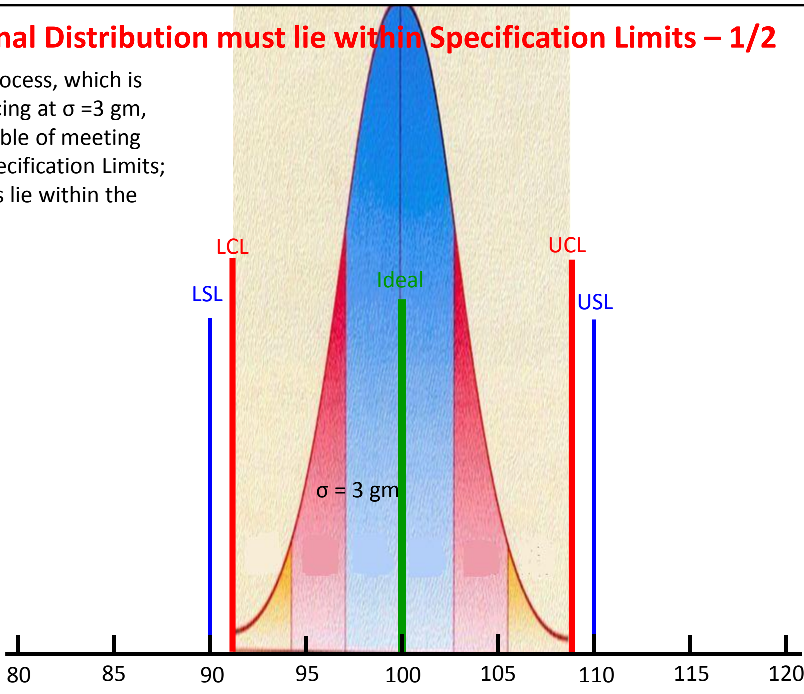
Normal Distribution must lie within Specification Limits – 1/2

This process, which is producing at σ =3 gm, is capable of meeting the Specification Limits; the CLs lie within the SLs



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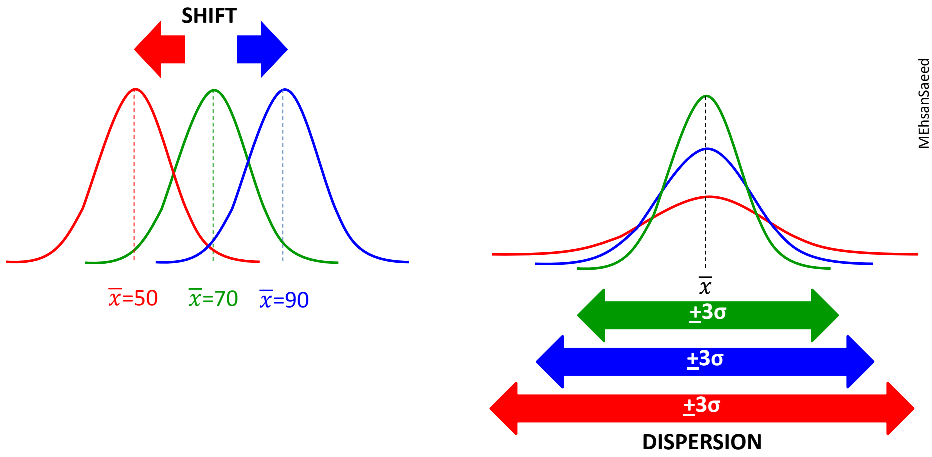
Normal Distribution – Shift & Dispersion



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Shift:

- Process shifts either side of Mean. **$\bar{x}$** changes, **σ** same

Dispersion:

- Dispersion of process across the Mean changes. **$\bar{x}$** same, **σ** changes

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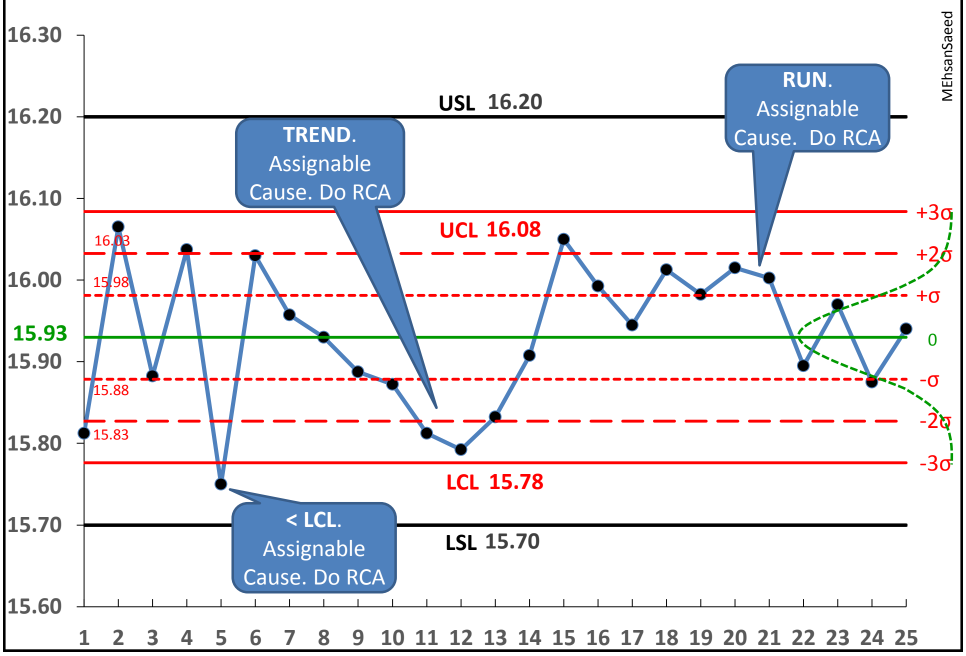
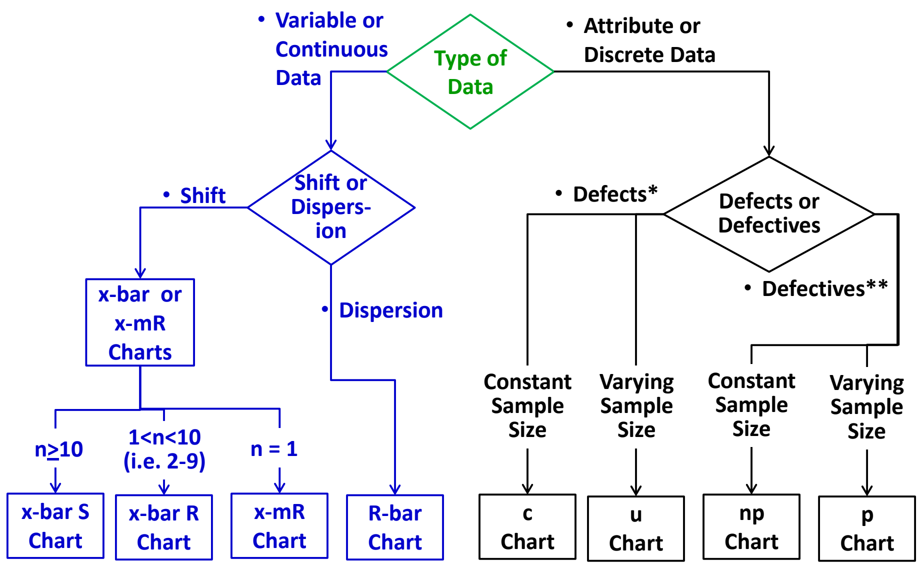
Typical Control Chart



Which Chart for What?



*Defects per Unit

** Units Defective due to One or More Defects

x-bar-S & x-bar-R Charts … 1/4

Used when *x=observation; S = Standard Deviation*

- there is a shift in the central tendency, i.e. the process mean, and more than one observation are available per time period (sample)
- **x-bar R** used when sample size is 2-9 and and **x-bar S** when sample size $\geq$ 10

Example 1:

Masonry blocks arrive at the project site by truckloads and are placed into a pile. A sample of 5 blocks is taken from the pile and tested for breaking strength (in tonne). 6 truckloads have so far arrived. Develop x-bar S and x-bar R charts for 3-sigma process if the testing observations are as follows:

Sample/Pile #	Block 1	Block 2	Block 3	Block 4	Block 5
1	0.96	0.80	1.00	0.92	0.96
2	1.20	1.00	1.10	1.10	1.00
3	0.80	0.80	0.80	0.80	0.80
4	0.90	0.90	0.90	1.00	1.00
5	0.70	1.00	1.10	1.20	0.70
6	0.96	1.00	0.80	1.00	1.00

x-bar-S & x-bar-R Charts … 2/4

Chart Limits for x-bar S:

$$= \bar{\bar{x}} \pm z\sigma_{\bar{x}} = \bar{\bar{x}} \pm z\frac{\sigma}{\sqrt{n}}$$

where:

$\bar{\bar{x}}$ = mean of sample means, i.e. the population mean = **CL**

z = Quality standard, eg 3 sigma or more, or degree of assurance/confidence

$\sigma_{\bar{x}}$ = standard deviation of sample means = $\frac{\sigma}{\sqrt{n}}$

σ = standard deviation of population

n = sample size

Chart Limits for x-bar R:

$$= \bar{\bar{x}} \pm A_2\bar{R}$$

where:

$\bar{\bar{x}}$ = mean of sample means, i.e. the population mean = **CL**

A_2 = Constant dependent on n

$\bar{R}$ = Mean of Sample Ranges

n = sample size

x-bar-S & x-bar-R Charts … 3/4

Pile (Sample #)	Block 1	Block 2	Block 3	Block 4	Block 5	Sample Mean $\bar{x}$	Sample σ	Sample Range R
1	0.96	0.80	1.00	0.92	0.96	0.928	0.077	0.200
2	1.20	1.00	1.10	1.10	1.00	1.080	0.084	0.200
3	0.80	0.80	0.80	0.80	0.80	0.800	0.000	0.000
4	1.10	1.00	0.90	1.00	1.00	1.000	0.071	0.200
5	0.80	1.00	1.10	0.90	0.70	0.900	0.158	0.400
6	0.96	0.90	0.80	1.00	0.90	0.912	0.076	0.200
					Mean	0.937*	0.078**	0.200

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x-bar S Chart

Here:

$\bar{\bar{x}}$ = 0.937*

z = 3 n =5

σ = 0.1184436

$\sigma_{\bar{x}}$ = 0.1184436/$\sqrt{5}$= 0.053 (using the formula $\frac{\sigma}{\sqrt{n}}$), or 0.078**

Chart Limits

$= \bar{\bar{x}} \pm z\sigma_{\bar{x}}$

= 0.937* ± 3 x 0. 053(or) 0.937* ± 3 x 0.078**

= 1.10, 0.78 tonne (or) 1.17, 0.70 tonne

x-bar R Chart

Here:

$\bar{\bar{x}}$ = 0.937*

A_2 = 0.577 *(from the tables)*

$\bar{R}$ = 0.200∸

n = 5

Chart Limits

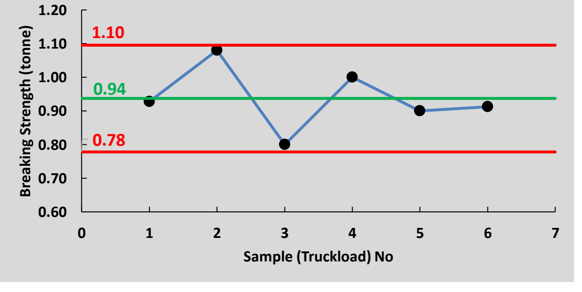
= $\bar{\bar{x}} \pm A_2\bar{R}$

=0.937* ± 0.577 x 0.200∸

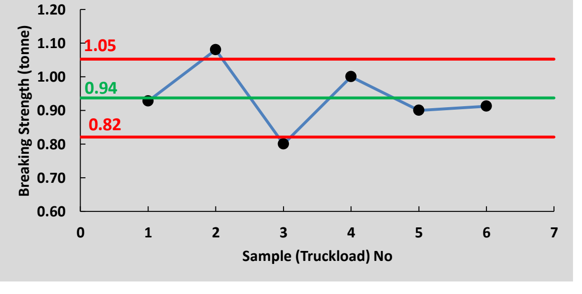
= 1.05, 0.82 tonne

x-bar-S & x-bar-R Charts … 4/4

x-bar S Chart



x-bar R Chart



The sample size was 5 for which the right chart was x-bar R. The two charts were drawn on the same sample size to bring out the differences which can be clearly noticed in UCL and LCL

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R-Charts … 1/2

R = Sample Range

Used when

there is a shift in the dispersion, or variability, of data, as indicated by the samples Ranges

Example 2:

Same as the for the x-bar S and x-bar R charts, in Example 1. Develop the R Chart for the process

Sample/Pile #	Block 1	Block 2	Block 3	Block 4	Block 5
1	0.96	0.80	1.00	0.92	0.96
2	1.20	1.00	1.10	1.10	1.00
3	0.80	0.80	0.80	0.80	0.80
4	0.90	0.90	0.90	1.00	1.00
5	0.70	1.00	1.10	1.20	0.70
6	0.96	1.00	0.80	1.00	1.00

R-Charts … 2/2

Chart Limits for R Chart:

$CL = \overline{R}$ where: $\overline{R}$ = mean of samples Ranges

$UCL = D_4\overline{R}$ $LCL = D_3\overline{R}$ D_3, D_4 = Constants dependent on sample size n

Taking the same example of blocks:

Pile (Sample #)	Block 1	Block 2	Block 3	Block 4	Block 5		Sample Range **R**
1	0.96	0.80	1.00	0.92	0.96		0.200
2	1.20	1.00	1.10	1.10	1.00		0.200
3	0.80	0.80	0.80	0.80	0.80		0.000
4	0.90	0.90	0.90	1.00	1.00		0.010
5	0.70	1.00	1.10	1.20	0.70		0.500
6	0.96	1.00	0.80	1.00	1.00		0.200
						Mean $\overline{R}$=	**0.200**

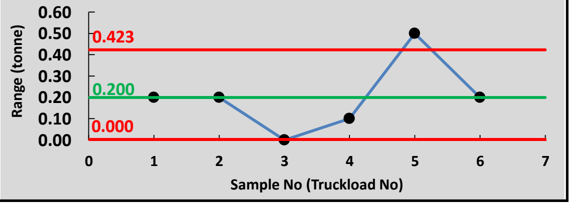
Chart Limits for R Chart:

CL = $\overline{R}$ = 0.200

D_4 = 2.114, D_3 = 0.000 *(tables)*

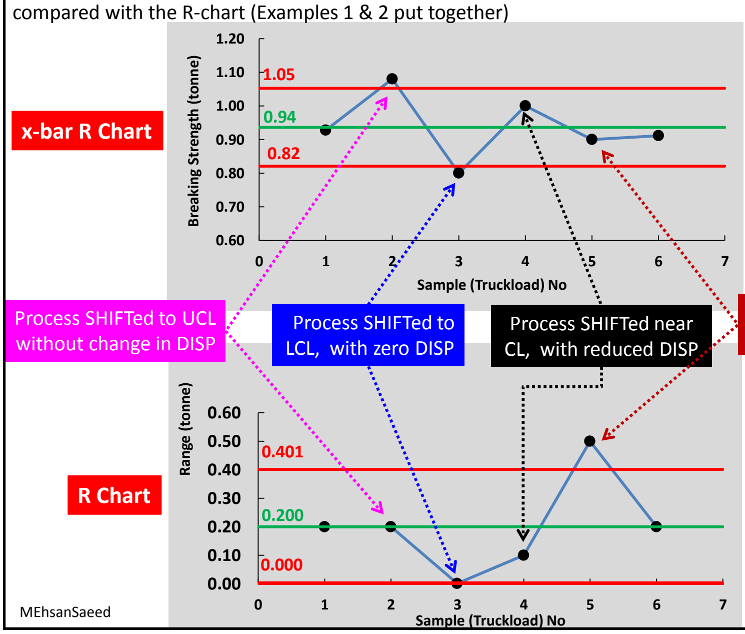
UCL = 2.114 x 0.200 = 0.423

LCL = 0.000 x 0.200 = 0.000



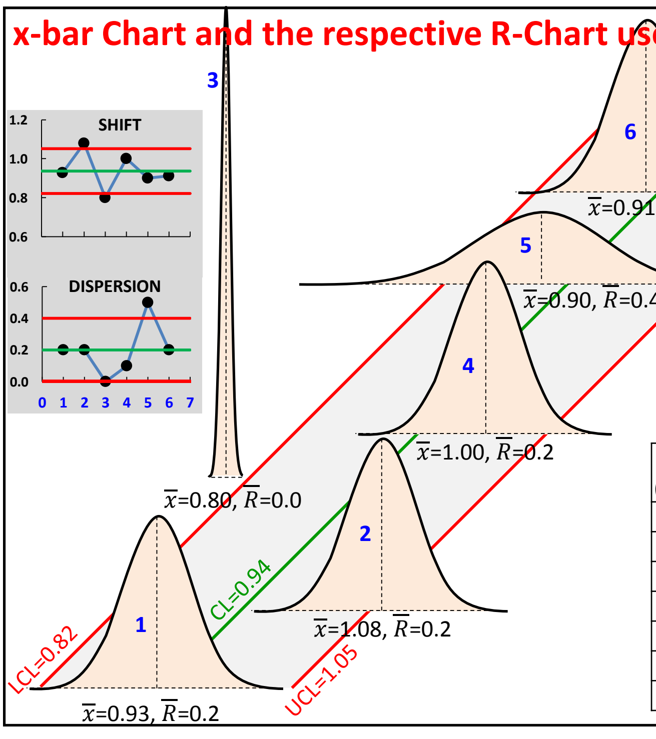
x-bar Chart and the respective R-Chart used in Unison - 1/2

For any process, the x-bar chart and the R-chart are always seen in combo, to fully appreciate how the process is going. In this example, the relevant chart was x-bar R (the sample size being 4) which will now be compared with the R-chart (Examples 1 & 2 put together)



- Process SHIFTING, alternating either side of the CL; process out of control at one point
- No idea how the DISPERSION is doing, i.e. the status of the std dev
- No idea if the process is SHIFTING on any side of the mean
- Truckloads 1,2 & 6 presented a steady DISP (std dev). DISP Zero/Lower in the 3rd and 4th ; DISP out of cont in 5th

x-bar Chart and the respective R-Chart used in Unison - 2/2



Pile (Sample #)	Sample Mean $\bar{x}$	Sample Range **R**
1	0.93	0.2
2	1.08	0.2
3	0.80	0.0
4	1.00	0.2
5	0.90	0.4
6	0.91	0.2
Mean	0.94	0.2

x-bar-S … Another Example - 1/2

Example 3:

On a production line, a sample of 10 items is taken after every hour and the items weighed. The results of last 9 inspections are tabulated. Develop an x-bar S chart if the plant is producing a 3-sigma quality level.

Sample #	Weight (gm)										
	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8	Item 9	Item 10	Sample Mean ($\bar{x}$)
1	80.30	86.90	108.00	80.30	86.90	108.00	80.30	86.90	108.00	86.90	91.25
2	99.40	89.50	96.40	99.40	89.50	96.40	99.40	89.50	96.40	89.50	94.54
3	95.10	95.90	85.30	95.10	95.90	85.30	95.10	95.90	85.30	95.90	92.48
4	99.00	123.90	100.60	99.00	123.90	100.60	99.00	123.90	100.60	123.90	109.44
5	97.10	98.60	107.70	97.10	98.60	107.70	97.10	98.60	107.70	98.60	100.88
6	97.40	105.50	104.50	97.40	105.50	104.50	97.40	105.50	104.50	105.50	102.77
7	97.90	106.00	95.60	97.90	106.00	95.60	97.90	106.00	95.60	106.00	100.45
8	81.60	99.90	101.10	81.60	99.90	101.10	81.60	99.90	101.10	99.90	94.77
9	90.80	90.10	95.10	90.80	90.10	95.10	90.80	90.10	95.10	90.10	91.81
										Mean of Sample Means ($\bar{\bar{x}}$) =	**97.60**
										σ =	**9.1**

UCL, LCL = $\bar{\bar{x}} \pm z\sigma_{\bar{x}} = \bar{\bar{x}} \pm z\frac{\sigma}{\sqrt{n}}$

= $97.6 \pm 3 \times \frac{9.1}{\sqrt{10}}$ = 106.23, 88.97 gm

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x-bar-S … Another Example - 2/2

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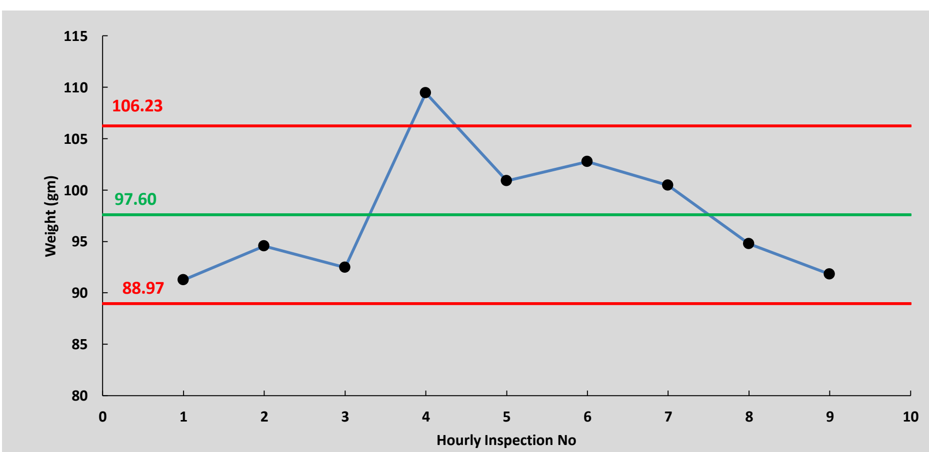
UCL, LCL = $\bar{\bar{x}} \pm z\sigma_{\bar{x}} = \bar{\bar{x}} \pm z\frac{\sigma}{\sqrt{n}}$

= $97.6 \pm 3 \times \frac{9.1}{\sqrt{10}}$ = 106.23, 88.97 gm



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3 Sigma Constants for x-bar & R-Charts

Sample Size (n)	X-bar Chart Constants A2	X-bar Chart Constants A3	R Chart Constants D3	R Chart Constants D4
1				
2	1.880	2.659	0.000	3.267
3	1.023	1.954	0.000	2.574
4	0.729	1.628	0.000	2.282
5	0.577	1.427	0.000	2.114
6	0.483	1.287	0.000	2.004
7	0.419	1.182	0.076	1.924
8	0.373	1.099	0.136	1.864
9	0.337	1.032	0.184	1.816
10	0.308	0.975	0.223	1.777
11	0.285	0.927	0.256	1.744
12	0.266	0.886	0.283	1.717
13	0.249	0.850	0.307	1.693
14	0.235	0.817	0.328	1.672
15	0.223	0.789	0.347	1.653
16	0.212	0.763	0.363	1.637
17	0.203	0.739	0.378	1.622
18	0.194	0.718	0.391	1.608
19	0.187	0.698	0.403	1.597
20	0.180	0.680	0.415	1.585
21	0.173	0.663	0.425	1.575
22	0.167	0.647	0.434	1.566
23	0.162	0.633	0.443	1.557
24	0.157	0.619	0.451	1.548
25	0.153	0.606	0.459	1.541

Difference between p/np and c/u Charts

p/np Charts

- deal with Defectives (non-conformant items)
- need sample size and number of defectives to be compared

c/u Charts

- deal with Defects (non-conformities)
- need number of defects only

Example

Situation 1: 2 items fail QC because of 4 faults on one & 5 on the other

p-Chart → There are 2 defectives (non-conformant items)

Situation 2: An item fails QC because of 4 faults

c-Chart → There are 4 defects (non-conformities)

	Defects	Defectives
Constant Size of Sample or Sub-Group	u	p
Variable Size of Sample or Sub-Group	c	np

p & np Charts

p=proportion, np=number proportion

Used when:

- there is a requirement to monitor and control the number, or proportion, of defectives (defective units) in a sample, as follows:
  - Defective or not Defective
  - Good or Bad
  - Broken or Not Broken
- **p Chart** used when the sample size is varying; therefore, proportion of defectives is considered
- **np Chart** is used when the sample size is constant; so number of defectives, rather than their proportion, is considered
- Examples: a carton with an x number of broken eggs, a packet with an x number of broken tiles; number of samples out of 20 with x,y,z number of nonconforming cables, number of packets with x,y,z number of nonconforming floppy disks when testing a samples of 200 for 25 trials etc

c & u Charts … 1/2

- Used when there is a requirement to monitor and control the **number** of defects (actual or proportional) in an item or in a measure, as follows:
  - Number of dents (actual or proportional) per item
  - Number of complaints (actual or proportional) per unit of time (hour, month, year etc)
  - Number of tears (actual or proportional) per unit of area (square foot, square meter)
  - Number of voids (actual or proportional) per inspection unit in injection molding or casting processes
  - Number of discrete components (actual or proportional) that must be re-soldered per printed circuit board
  - Number of product returns (actual or proportional) per day
- Examples: Actual or proportional number of:- bacteria in a petri dish, barnacles on the bottom of a boat, complaints from the customers, blemishes on auto body observed for 30 samples, imperfections in bond paper – by area inspected and number of imperfections

c & u Charts … 2/2

- **c Chart** used when the sample size is constant; therefore, actual number of defects is considered
- **u Chart** is used when the sample size is varying; so proportion of defects, rather than their actual number, is considered

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c-Chart … 1/2

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Example 4:

A teacher wishes to monitor and control the class attendance. He records the number of absentees over an academic week and the result is Mon: 5, Tue:3, Wed:3, Thu:2, Fri:4. The number of students on roll on these five days remained 40. Develop a control chart.

Explanation: This is a case of defects (absentees) in a unit (class). The sample size (class strength is fixed). ∴ Correct Chart is **c**

Day	Class Strength i.e. Sample Size (n)	Number of Absentees i.e. Defaults/ Defects (d)
Mon	40	6
Tue	40	4
Wed	40	5
Thu	40	3
Fri	40	5

Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$

where

$\bar{c}$ = CL & mean number of defects

z = Quality standard, eg 3 sigma or more, or degree of confidence

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c-Chart … 2/2

Day	Sample Size (n)	Defaults/ Defects (d)	Prop Defaults (d/n)	LCL	UCL
M	40	6	Not Reqd	-2.0	10.0
T	40	4		-2.0	10.0
W	40	5		-2.0	10.0
Th	40	3		-2.0	10.0
F	40	5		-2.0	10.0
∑	200	20			
$\bar{c}$	23/5 = 4.6 absentees per day				

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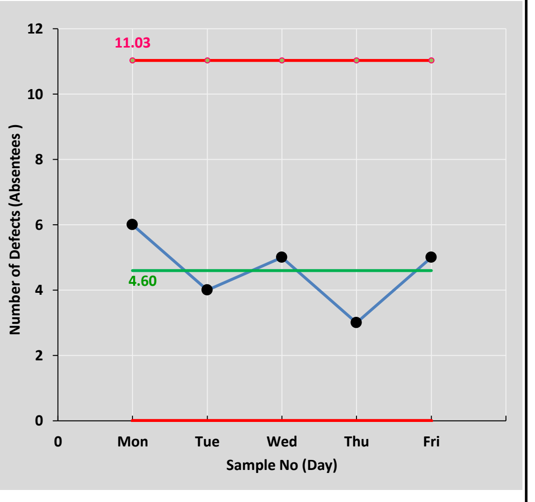

Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$

Here, the SD $\sqrt{\bar{c}}$ is constant $\therefore$ **LCL** &**UCL** for all observations are also constant, giving straight lines $\bar{c} = \frac{23}{5} = 4.6$, z=3

UCL, LCL = 4.6 $\pm$ 3$\sqrt{4.6}$ = 11.03 → 11, -1.83 → 0

u-Chart … 1/2

Example 5:

A teacher wishes to monitor and control the class attendance. He records the number of absentees over an academic week and the result is Mon: 5, Tue:3, Wed:3, Thu:2, Fri:4. The number of students on roll on these five days were 40, 42, 42, 38, 38. Develop a control chart.

Explanation: This is a case of defects (absentees) in a unit (class). The sample size (class strength is variable). $\therefore$ Correct Chart is **u**

Day	Class Strength i.e. Sample Size (n)	Number of Absentees i.e. Defaults/ Defects (d)
Mon	40	5
Tue	42	3
Wed	42	3
Thu	38	2
Fri	38	4

Cont Limits = $\bar{u} \pm z\sqrt{\frac{\bar{u}}{n}}$

where

$\bar{u}$ = CL & mean proportional number of defects

z = Quality standard, eg 3 sigma or more, or degree of confidence

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u-Chart … 2/2

Day	Sample Size (n)	Defaults/ Defects (d)	Prop Defaults (d/n)	LCL	UCL
M	40	6	0.092	-0.006	0.166
T	42	3	0.080	-0.005	0.165
W	42	4	0.050	-0.005	0.165
Th	38	3	0.069	-0.004	0.164
F	38	4	0.090	-0.005	0.165
∑	200	20			
$\overline{u}$	20/200 = **0.10**				

Cont Limits = $\overline{u} \pm z\sqrt{\frac{\overline{u}}{n}}$

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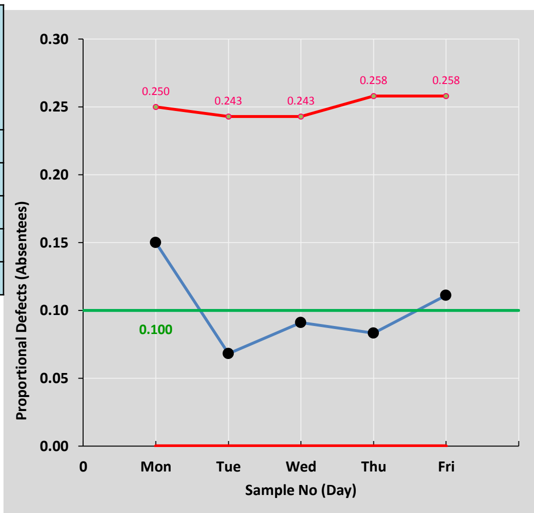

In each case (for each day), the sample size n varies. ∴ the standard deviation $\sqrt{\frac{\overline{u}}{n}}$, and hence **LCL** &**UCL** for each case vary, and form wiggly lines

np-Chart … 1/2

Example 6: An HOD wishes to monitor & control the class attendance in his Dept. He selects the 8 BBA classes and observes the number of absentees over the period of time. The results are BBA-1: 20, BBA-2: 22, BBA-3: 28, BBA-4: 22, BBA-5: 27, BBA-6: 20, BBA-7: 18 & BBA-8: 21. Assuming that the strength of each class is 40, develop a control chart.

Explanation: This is a case of defectives (classes) with defects (absentees). The sample size (class strength) is fixed ∴ Correct Chart is **np**

Cont Limits = $n\overline{p} \pm z\sigma_{np} = n\overline{p} \pm z\sqrt{n\overline{p}(1-\overline{p})}$

where

$\overline{p}$ = CL & mean of sample proportion defectives = $\frac{total\ defectives}{total\ observations}$

z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

σ_{np}= Std Dev of the Average Proportion Defective = $\sqrt{n\overline{p}(1-\overline{p})}$

n = mean sample size

np-Chart … 2/2

Defective BBA-	Sample (Class) Size (n)	Def-ects (D)	(Prop Def-ects D/n)	LCL	UCL
1	40	20	Not Reqd	12.82	31.68
2	40	22		12.82	31.68
3	40	28		12.82	31.68
4	40	22		12.82	31.68
5	40	27		12.82	31.68
6	40	20		12.82	31.68
7	40	18		12.82	31.68
8	40	21		12.82	31.68
∑	320	178			
$\bar{p}$	178/320 = 0.556				

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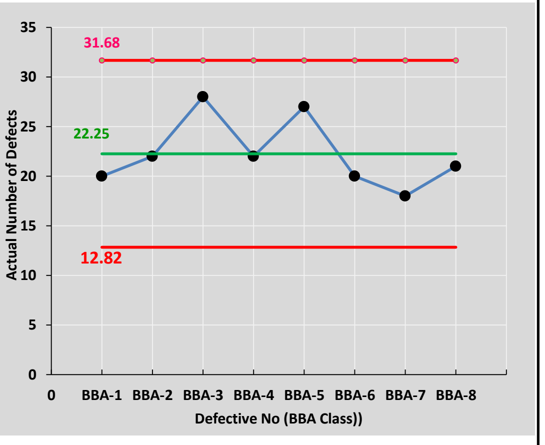


Cont Limits $= n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1-\bar{p})}$

In each case (for each class), sample size n is the same. $\therefore$ the SD $\sqrt{n\bar{p}(1-\bar{p})}$, and hence **LCL** &**UCL** for each case are also constant, giving straight lines

UCL, LCL = 40x0.556$\pm$3$\sqrt{40\text{x}0.556(1-0.556)}$ = 31.68 → 32, 12.82 → 13

p-Chart … 1/2

Example 7: An HOD wishes to monitor & control the class attendance in his Dept. He selects the 8 BBA classes and observes the number of absentees over the period of time. The results are BBA-1: 20, BBA-2: 22, BBA-3: 28, BBA-4: 22, BBA-5: 27, BBA-6: 20, BBA-7: 18 & BBA-8: 21. The class strength was 40, 42, 36, 44, 41, 35, 44 & 43 respectively.

Explanation: This is a case of defectives (classes) with defects (absentees). The sample size (class strength) is variable $\therefore$ Correct Chart is **p**.

Cont Limits $= \bar{p} \pm z\sigma_p = \bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

where

$\bar{p}$ = CL & mean of sample proportion defectives = $\frac{total\ defectives}{total\ observations}$

z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

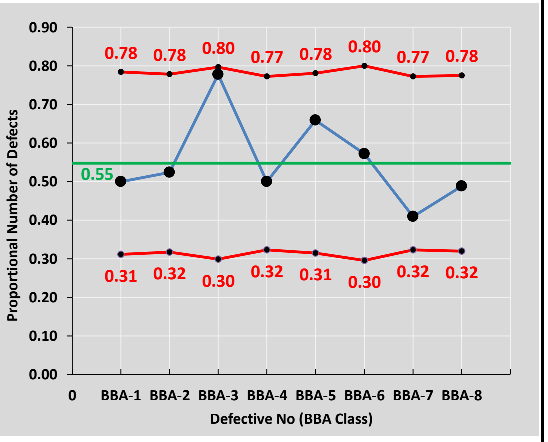
σ_p= Std Dev of the Average Proportion Defective = $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

n = mean sample size

MEhsanSaeed

p-Chart … 2/2

Defective BBA-	Sample (Class) Size (n)	Def-ects (D)	Prop Def-ects (D/n)	LCL	UCL
1	40	20	0.14	-0.10	0.30
2	42	22	0.10	-0.10	0.30
3	36	28	0.06	-0.11	0.31
4	44	22	0.10	-0.10	0.30
5	41	27	0.06	-0.12	0.32
6	35	20	0.15	-0.10	0.30
7	44	18	0.17	-0.11	0.31
8	43	21	0.11	-0.11	0.31
∑	325	178			
$\overline{p}$	178/325 = 0.56				



Cont Limits = $\overline{p} \pm z\sigma_p = \overline{p} \pm z\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$

In each case (for each class), the sample size n varies. ∴the SD $\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$, and hence **LCL** & **UCL** for each case vary, and form wiggly lines

c-Chart … 1/2

Example 8:

100 workers are transported to the project site daily in buses. The number of workers missing the buses in the last 10 days has been observed to be 9,8,5,7,9,8,9,4,9,12. Develop a 3-Sigma c-Chart for the defaulting workers

Day	Number of Workers i.e. Sample Size (n)	Those who missed the buses i.e. Defaults/ Defects (d)
1	100	9
2	100	8
3	100	5
4	100	7
5	100	9
6	100	8
7	100	9
8	100	4
9	100	9
10	100	12

Cont Limits = $\overline{c} \pm z\sqrt{\overline{c}}$

where
$\overline{c}$ = CL & mean number of defects
z = Quality standard, eg 3 sigma or more, or degree of confidence

c-Chart … 2/2

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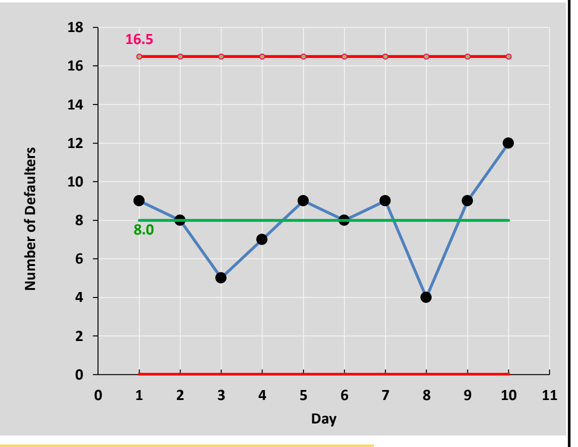
Day	Defaults/ Defects	Prop Def	LCL	UCL
1	9	Not Reqd	16.5	-0.5
2	8		16.5	-0.5
3	5		16.5	-0.5
4	7		16.5	-0.5
5	9		16.5	-0.5
6	8		16.5	-0.5
7	9		16.5	-0.5
8	4		16.5	-0.5
9	9		16.5	-0.5
10	12		16.5	-0.5
∑	80			
$\bar{c}$	80/10 =8.0 per day			



Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$

Here, the SD $\sqrt{\bar{c}}$ is constant $\therefore$ **LCL** &**UCL** for all observations are also constant, giving straight lines $\bar{c} = \frac{80}{10} = 8.0$, z=3

UCL, LCL = 8.0 $\pm$ 3$\sqrt{8.0}$ = 16.5 → 17, -0.5 → 0

u-Chart … 1/2

Example 9:

A construction company transports its workers to project site in buses. The number of workers on the company's register over the last 10 days, and those who missed the buses are as tabulated. Develop a 3-Sigma u-Chart for the defaulting workers

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Day	Number of Workers i.e. Sample Size (n)	Those who missed the buses i.e. Defaults/ Defects (d)
1	98	9
2	100	8
3	100	5
4	102	7
5	100	9
6	99	8
7	99	9
8	100	4
9	100	9
10	102	12

Cont Limits = $\bar{u} \pm z\sqrt{\frac{\bar{u}}{n}}$

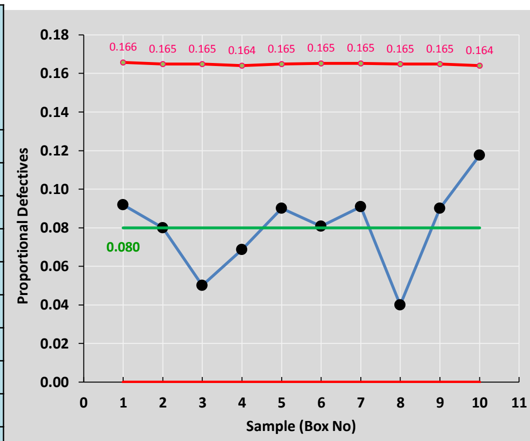
where

$\bar{u}$ = CL & mean proportional number of defects

z = Quality standard, eg 3 sigma or more, or degree of confidence

u-Chart … 2/2

Day	Sample Size (n)	Defaults/ Defects (d)	Prop Defaults (d/n)	LCL	UCL
1	98	9	0.092	-0.006	0.166
2	100	8	0.080	-0.005	0.165
3	100	5	0.050	-0.005	0.165
4	102	7	0.069	-0.004	0.164
5	100	9	0.090	-0.005	0.165
6	99	8	0.081	-0.005	0.165
7	99	9	0.091	-0.005	0.165
8	100	4	0.040	-0.005	0.165
9	100	9	0.090	-0.005	0.165
10	102	12	0.118	-0.004	0.164
∑	1,000	80			
$\bar{u}$	80/1,000 =**0.08**				



Cont Limits = $\bar{u} \pm z\sqrt{\frac{\bar{u}}{n}}$

In each case (for each day), the sample size n varies. ∴ the standard deviation $\sqrt{\frac{\bar{u}}{n}}$, and hence **LCL** &**UCL** for each case vary, and form wiggly lines

np-Chart … 1/2

Example 10:
On a large construction project, 10 boxes of electrical switches have arrived. Each box has 1,000 switches. Randomly, the procurement manager picks up 20 switches each from the 10 boxes. He finds 3, 3, 4, 2, 1, 3, 2, 3, 2 & 1 switches defective, in the ten boxes. Develop a 3-sigma np-Chart for the sampling done.

Cont Limits = $n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1-\bar{p})}$

where

$\bar{p}$ = CL & mean of sample proportion defectives = $\frac{total\ defectives}{total\ observations}$

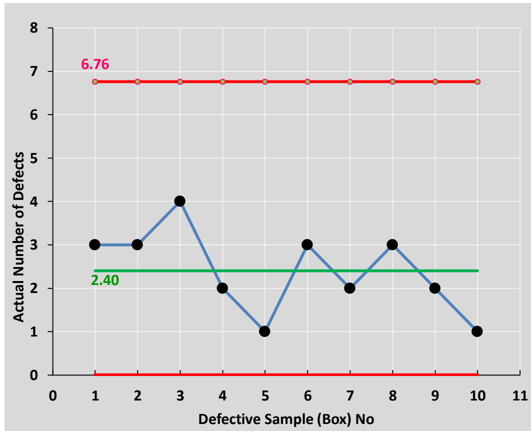
z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

σ_{np}= Std Dev of the Average Proportion Defective = $\sqrt{n\bar{p}(1-\bar{p})}$

n = mean sample size

np-Chart … 2/2

Def Box No	Sample Size (n)	Defects (d)	(Prop defects (d/n))	LCL	UCL
1	20	3		-1.96	6.76
2	20	3		-1.96	6.76
3	20	4		-1.96	6.76
4	20	2		-1.96	6.76
5	20	1	Not	-1.96	6.76
6	20	3	Reqd	-1.96	6.76
7	20	2		-1.96	6.76
8	20	3		-1.96	6.76
9	20	2		-1.96	6.76
10	20	1		-1.96	6.76
∑	200	24			
$\bar{p}$	24/200 = 0.12				



Cont Limits = $n\bar{p} \pm z\sigma_{np}$ = $n\bar{p} \pm z\sqrt{n\bar{p}(1-\bar{p})}$

In each case (for each box), the sample size n is the same. ∴ the SD $\sqrt{n\bar{p}(1-\bar{p})}$, and hence **LCL** & **UCL** for each case are also constant, giving straight lines

UCL, LCL = 20x0.12$\pm$3$\sqrt{20\text{x}0.12(1-0.12)}$ = 6.76 → 7, -1.96 → 0

p-Chart … 1/2

Example 11:
On a large construction project, 10 boxes of electrical switches have arrived. Each box has 1,000 switches. Randomly, the procurement manager picks up 22, 20, 18, 18, 18, 20, 15, 18, 18 & 20 switches from the 10 boxes. He finds 3, 2, 1, 2, 1, 3, 2, 1, 2 & 3 switches defective, respectively, in the ten boxes. Develop a 3-sigma p-Chart for the sampling done.

Cont Limits = $\bar{p} \pm z\sigma_p = \bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

where

$\bar{p}$ = CL & mean of sample proportion defectives = $\frac{total\ defectives}{total\ observations}$

z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

σ_p= Std Dev of the Average Proportion Defective = $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

n = mean sample size

p-Chart … 2/2

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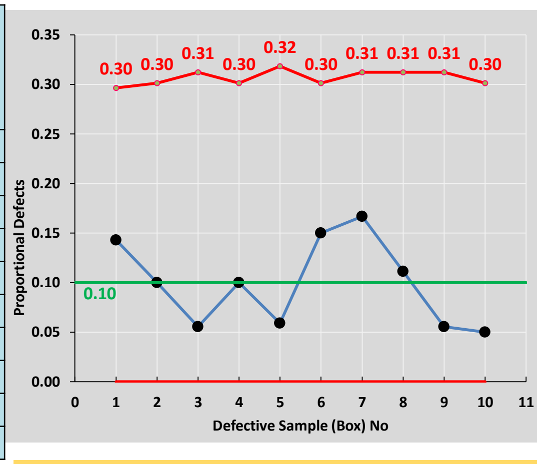
Def Box No	Sample Size (n)	Defects (d)	Prop Def-ects (d/n)	LCL	UCL
1	21	3	0.14	-0.10	0.30
2	20	2	0.10	-0.10	0.30
3	18	1	0.06	-0.11	0.31
4	20	2	0.10	-0.10	0.30
5	17	1	0.06	-0.12	0.32
6	20	3	0.15	-0.10	0.30
7	18	3	0.17	-0.11	0.31
8	18	2	0.11	-0.11	0.31
9	18	1	0.06	-0.11	0.31
10	20	1	0.05	-0.10	0.30
∑	190	19			
$\bar{p}$	19/190=0.100				



Cont Limits = $\bar{p} \pm z\sigma_p = \bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

In each case (for each box), the sample size n varies. ∴ the SD $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$, and hence **LCL** & **UCL** for each case vary, and form wiggly lines